DEPARTMENT OF ECONOMICS — MAY 5, 2023
SAN JOSE STATE UNIVERSITY — 6:00 P.M. TO 9:30 P.M.
MASTER'S COMPREHENSIVE EXAMINATION — PROCTOR: LIU

INSTRUCTIONS:

1. Answer ONLY the specified number of questions from the options provided in each section. Do not answer more than the required number of questions. Each section takes one hour.

2. Your answers must be on the paper provided. No more than one answer per page. Do not answer two questions on the same sheet of paper.

3. If you use more than one sheet of paper for a question, write "Page 1 of 2" and "Page 2 of 2."

4. Write ONLY on one side of each sheet. Use only pen. Answers in pencil will be disqualified.

5. Write **------ END -----** at the end of each answer.

6. Write your exam identification number in the upper right-hand corner of each sheet of paper.

7. Write the question number in the upper right-hand corner of each sheet of paper.

Section 1: Microeconomic Theory—Answer Any Two Questions.

1A. (Econ 201) Assume that the market (inverse) demand function for a homogeneous good is $P(Q) = a - bQ$, where a and b are positive constants, and Q is the total quantity of the good on the market. There are two active firms in this market. Firm 1 has a constant marginal cost of c: $C_1(q_1) = cq_1$. Firm 2 has a constant marginal cost of d: $C_2(q_2) = dq_2$. Assume that the firms compete by setting quantities. Calculate the Nash equilibrium and the corresponding market price.

1B. (Econ 201) A firm produces output q using a production function $q = \frac{1}{8} L^{\frac{1}{2}} K^{\frac{1}{2}}$ where L is labor, and K is capital. Capital K is fixed at a level $\underline{K} = 16$, and its price is $r = 10$. Denote the price of labor as w, and the price of output as p.

i. For $w = 2$, calculate the firm's short-run total cost function *STC(q)*, short-run average cost function *SAC(q)*, and short-run marginal cost function *SMC(q)*?
ii. If this firm faces the following downward-sloping inverse demand function $p = a - bq$ (*a* and *b* are both positive constants), construct the firm's profit function? What is the optimal level of output q^* ?
iii. At the input prices *(r,w)*, calculate the firm's long-run total cost function *LTC(q),* long-run average cost function *LAC(q)* , and Long-run marginal cost function *LMC(q)*?

1C. (ECON 104)

(a) Consider the utility maximization problem

$$\text{Max } U(x, y) = \sqrt{x} + y \text{ subject to } x + 4y = 100.$$

Solve the problem by transforming it into an <u>*unconstrained*</u> optimization problem with one variable.

(b) Solve the same utility maximization problem

$$\text{Max } U(x, y) = \sqrt{x} + y \text{ subject to } x + 4y = 100$$

using the Lagrange method, i.e. find the quantities demanded of the two goods. Show that the Lagrange method leads to the same solution.

(c) Suppose income increases from 100 to 101. What is the exact increase in the optimal value of $U(x, y)$? Compare with the value found in (b) for the Lagrange multiplier.

(d) Interpret the Lagrange multiplier.

(e) Suppose we change the budget constraint to $px + qy = m$, but keep the same utility function. Derive the quantities demanded of the two goods if $m > q^2/4p$.

(over)